Zero-Delay Joint Source-Channel Coding in the Presence of Interference Known at the Encoder

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Abstract—Zero-delay transmission of a Gaussian source is considered over an additive white Gaussian noise (AWGN) channel in the presence of an additive Gaussian interference signal. The mean squared error (MSE) distortion is minimized under an average power constraint assuming that the interference signal is known causally at the transmitter. Optimality of simple uncoded transmission does not hold in this setting due to the presence of the known interference signal, and various non-linear transmission schemes are proposed. In particular, interference concentration (ICO) and one-dimensional lattice (1DL) strategies are studied. It is shown that non-uniform quantization of the interference signal for ICO improves the performance.

I. INTRODUCTION

Emerging mobile cyber-physical systems (CPS) require coordination and communication among autonomous network agents as well as remote monitoring of critical network parameters for optimal control and resource allocation. In many scenarios, local system parameters are measured and reported by sensors to other network agents over noisy communication links. One important aspect of communication for CPSs is the strict latency constraints: the monitoring and control of the network has to be near real-time. Such extreme latency constraints preclude the use of long codes either for compression or communication of measured parameters. Accordingly, in this paper we study zero-delay transmission of system parameters over wireless channels.

When transmitting a Gaussian source over an additive white Gaussian noise (AWGN) channel it is well-known that the zero-delay constraint does not lead to any performance loss in terms of the end-to-end mean-squared error (MSE) distortion; and the optimal Shannon lower bound can be achieved by simple linear encoding [1]. However, the optimality of linear encoding is sensitive to the matching between the source and channel distributions as well as the distortion measure; and characterization of the optimal transmission strategy is challenging in general, and remains an open problem in most cases [2]–[8].

In this paper we consider zero-delay transmission of a Gaussian source over an AWGN channel in the presence of an additive interference causally known at the transmitter, known as the dirty-tape channel. The capacity of the dirty-tape channel was first studied by Shannon [9], who characterized the capacity using the so-called Shannon strategies. On the other hand, the channel model when the interference is known non-causally at the transmitter is known as the dirty-paper channel. The capacity of the dirty-paper channel was characterized by Gelfand and Pinsker in [10], and it was later shown in [11] that, in the Gaussian setting, the capacity of the dirty-paper channel is equal to the one without interference.

On the contrary, despite Shannon’s single-letter characterization, there is no closed-form capacity expression for the dirty-tape channel even in the Gaussian setting. Willems proposed interference concentration (ICO) for the Gaussian dirty-tape channel in [12]. This scheme is based on the idea of canceling the interference by giving a structure to it. Willems showed that, by quantizing the interference at the encoder, it is possible to mitigate it at the receiver by proper power allocation between the interference quantization error and the channel input signal, which is uniformly distributed over the quantization region. More recently, Erez et al. [13] consider using inflated lattice strategies for the Gaussian dirty-tape channel in [12]. They show that the rate loss of their coding scheme with respect to no interference, which can be achieved in the case of dirty-paper channel [11], is not more than 0.254 bits per channel use in the asymptotic high SNR regime. On the other hand, it is shown in [14] that the ICO scheme of Willems performs better than the inflated lattice based coding scheme in the low SNR regime. In [15], optimal mappings based on an iterative algorithm are proposed.

We remark here that, all of the above mentioned work focus on the channel coding problem. To the best of our knowledge, the zero-delay joint source channel coding (JSCC) problem over the dirty-tape channel has not been studied before. Note that in the zero-delay JSCC problem, Shannon’s source-channel separation theorem [16] does not apply; and hence, we can not directly use the above channel coding
results to evaluate the MSE performance. Instead we develop joint source-channel transmission techniques that are motivated by the strategies proposed in [12] and [13]. We consider ICO and one-dimensional lattice (1DL) schemes combined with nonlinear companders. While characterizing the optimal performance is elusive for this problem, we present numerical results comparing the performance of the proposed strategies, and provide some heuristics to improve them. In particular, we propose a counter-intuitive non-uniform quantization scheme in conjunction with ICO, which increases the average quantization error, and hence, the power used for interference concentration, but leads to a lower MSE since the transmitter can then use a compander with a larger dynamical range for the more likely interference states.

The rest of the paper is organized as follows: In Section II we introduce the system model. In Section III zero-delay transmission schemes under average power constraint are proposed. In Section IV numerical results are discussed, and in Section V we conclude the paper.

II. SYSTEM MODEL

We consider the transmission of a Gaussian source over an AWGN channel in the presence of an additive interference signal, which is causally known at the transmitter. The setup is illustrated in Figure 1. The memoryless Gaussian source samples \( \{V_i\} \) are independent and identically distributed (i.i.d.) with zero mean and variance \( \sigma_v^2 \), i.e., \( V_i \sim \mathcal{N}(0, \sigma_v^2) \). The interference signal is independent of the source samples and i.i.d. with a Gaussian distribution, \( S_i \sim \mathcal{N}(0, \sigma_s^2) \). The discrete memoryless channel output at time \( i \), \( Y_i \), is given by

\[
Y_i = X_i + S_i + W_i,
\]

where \( X_i \) is the channel input, \( S_i \) is the known Gaussian interference signal, and \( W_i \) is the additive i.i.d. Gaussian noise, \( W_i \sim \mathcal{N}(0, \sigma_w^2) \), independent of the source and the interference. The transmitter is required to transmit the source samples, \( V_i \), over the channel with minimum mean squared error (MMSE).

We denote the zero-delay encoding function at time \( i \) as \( X_i = h_i(V^i, S^i) \), where \( V^i \triangleq \{V_1, ..., V_i\} \), and at the receiver each source sample \( V_i \) needs to be estimated after the i-th transmission; hence, we have \( \hat{V}_i = g_i(Y^i) \), where \( g_i \) denotes the reconstruction function for the i-th source sample. Due to the i.i.d. nature of the source, interference and noise components, \( h_i(\cdot) \) and \( g_i(\cdot) \) can be simplified to \( X_i = b(V_i, S_i) \) and \( \hat{V}_i = g(Y_i) \), without loss of generality. For simplicity we remove the index \( i \) hereafter. The average power constraint is defined as below

\[
E[|h(V, S)|^2] \leq P. \tag{1}
\]

We note that, in our setting, due to the zero-delay constraint, causal and non-causal knowledge of the interference are equivalent. In other words, non-causal knowledge of the interference is useless, and the transmitter only uses the knowledge of the current value of the interference.

Our goal is to characterize the minimum achievable MSE, \( E[(V - \hat{V})^2] \), for given \( P \), \( \sigma_s \) and \( \sigma_n \) values.

III. ACHIEVABLE TRANSMISSION SCHEMES

In this section we introduce four different encoding schemes with increasing complexity. Later in Section IV we will compare and comment on the performances of these schemes.

A. Interference Cancelation (ICA)

The simplest way to communicate in the presence of known interference is to cancel the interference. In the interference cancellation (ICA) scheme, the transmitted signal \( X \) is a simple linear combination of the source realization \( V \) and the interference \( S \). The transmitter decides how much of the interference will be cancelled depending on the system parameters. We have

\[
X = aV + bS, \tag{2}
\]

where \( a \) and \( b \) are the coefficients to be determined. The channel input has to satisfy

\[
P \geq E[|X|^2] = a^2\sigma_v^2 + b^2\sigma_s^2. \tag{3}
\]

With MMSE estimation at the receiver, the achievable average distortion is found as

\[
D_{ICA} = \frac{\sigma_v^2}{1 + \frac{P - b^2\sigma_s^2}{(b+1)^2\sigma_s^2 + \sigma_n^2}}. \tag{4}
\]

The optimal \( b \) value that minimizes (4) is given by

\[
b^* = -\frac{P + \sigma_s^2 + \sigma_n^2 - \sqrt{(P + \sigma_n^2)^2 + \sigma_s^4 - 2\sigma_n^2(P - \sigma_n^2)}}{2\sigma_s^2}. \tag{5}
\]

The optimal value for \( a \) can be obtained from (4) and (5). The ICA scheme wastes part of the transmission power for interference cancellation; and thus, is expected to perform poorly especially in the low power regime. The analysis of the performance of the ICA scheme is relegated to Section IV. Next we will introduce alternative non-linear transmission strategies.

B. Interference Concentration (ICO)

This scheme is motivated by Willems’ ICO scheme for channel coding [12]. We combine the interference concentration idea with JSCC of a Gaussian signal under a peak power constraint (PPC) [3]. The interference signal \( S \) is concentrated to one of the pre-determined discrete points on the real line; that is, the interference is quantized, and the corresponding quantization noise is cancelled rather than cancelling the whole interference. Only the quantization index of the interference is received at the receiver. The transmitter superposes a compounded version of the source signal such that it is compressed into one quantization interval of the quantizer.

The source is clipped and mapped to the interval \([-\Delta/2, \Delta/2]\) as below

\[
T(v) = \begin{cases} 
\Delta, & v \geq \frac{\Delta}{2}, \\
\frac{v}{2}, & -\frac{\Delta}{2} \leq v < \frac{\Delta}{2}, \\
-\Delta, & v < -\frac{\Delta}{2}. 
\end{cases} \tag{6}
\]
We denote the probability density function (pdf) of transformed source \( T(V) \) by \( f_T \). The signal transmitted over the channel is given by

\[
x = T(v) - (s \mod \Delta),
\]

where \( s \mod \Delta \in [-\Delta/2, \Delta/2) \) corresponds to the quantization error, and is defined as

\[
s \mod \Delta = s - Q(s),
\]

where \( Q(s) \) is the nearest neighbor quantizer defined as below

\[
Q(s) \triangleq \Delta \cdot \left\lfloor \frac{s}{\Delta} + \frac{1}{2} \right\rfloor,
\]

where \( \lfloor . \rfloor \in \mathbb{N} \) is the nearest integer in the negative direction.

It can be seen from (8) that, in the ICO scheme \( s \) is concentrated to one of the quantization indices in \( \{ i\Delta : i \in \mathbb{Z} \} \). The power that the transmitter uses for interference concentration is equivalent to the average quantization noise of the interference signal.

In order to satisfy the power constraint, the power of the two components of the channel input \( X \) in (8) should be chosen properly. While the power allocated to interference concentration depends only on the value of \( \Delta \), the power of the compander component depends on both \( \Delta \) and \( \Delta_v \) parameters. While it is possible to characterize these power values in terms of \( \Delta \) and \( \Delta_v \) values, these are complex expressions involving infinite summations and error function; and hence, are omitted here. Since a closed-form expression for the input power is difficult to obtain, we will resort to numerical techniques to find the appropriate \( \Delta \) and \( \Delta_v \) parameters that satisfy the average power constraint.

The received signal is

\[
y = x + s + w
\]

\[
= T(v) - (s \mod \Delta) + s + w
\]

\[
= T(v) + Q(s) + w.
\]

Decoding is done based on MMSE estimation, whose output is found as below:

\[
\hat{V}_{ICO} = \frac{\sum_{i} p(q_i) \int_{-\infty}^{\infty} u f_Y(u) f_W(y - T(u) - q_i) du}{\sum_{i} p(q_i) \int_{-\infty}^{\infty} f_Y(u) f_W(y - T(u) - q_i) du},
\]

where \( q_i \triangleq i\Delta, i \in \mathbb{Z}, \) are the points to which the interference is concentrated. We can further expand (12) as in (7). Finally, the corresponding average distortion is evaluated as below

\[
D_{ICO} = \sum_{i \in \mathbb{Z}} p(q_i) \int_{w} \int_{v} (v - \hat{v}_{ICO})^2 f_W(w) f_Y(v) dv dw.
\]

C. Comparison of ICA and ICO in the Asymptotic Zero-Noise Regime

In order to illustrate the benefits of ICO over ICA we consider the asymptotic zero-noise regime, i.e., we assume that \( \sigma_n^2 \to 0 \). For ICA, one can see that, if \( P \geq \sigma_s^2 \) then the interference can be completely removed using part of the available power, and zero distortion is achieved in the limit as the noise disappears. On the other hand, when \( P < \sigma_s^2 \), the best achievable distortion is \( \sigma_s^2(\sigma_s^2 - P)/\sigma_n^2 \); that is, there is always residual distortion in the estimation even if there is no noise in the system.

On the other hand, one can show that zero distortion can be achieved by the ICO scheme in the asymptotic zero-noise regime, independent of the available power value. We will explain this fact intuitively. In the absence of noise, since the received signal is always within the quantization region of the interference signal, the quantization index can always be decoded correctly. Once the quantization index is known, the effect of interference can be completely removed. The remaining distortion is only from the companding of the source samples to squeeze them into the quantization region. By letting \( \Delta_v \) go to infinity we can reconstruct the source perfectly, and zero distortion can be achieved asymptotically. Note that, letting \( \Delta_v \to \infty \) also means that the input power constraint depends only on the value of \( \Delta \) in the limit.
These arguments show that the ICO scheme can provide significant improvements compared to ICA particularly when the interference is high while the noise in the system is low. In the following we provide two other techniques based on the idea of providing a structure to the interference. We will observe that these techniques will further improve the performance of the ICO scheme.

D. One Dimensional Lattice (1DL)

The idea of using a lattice structure for communication in the presence of known interference has been considered in [13] for the channel coding problem. Here we consider using a similar lattice structure for JSCC. The channel input for the 1DL scheme is given by

\[ x = (T(v) - s) \mod \Delta, \quad (14) \]

where \( T(v) \) is as defined in (9). In the 1DL scheme, the term \( T(v) - s \) is concentrated to one of the quantization points in \( \{i \cdot \Delta\}_{i=-\infty}^{\infty} \).

In order to satisfy the average power constraint, we need to characterize the pdf of \( X \). Define \( L_2 \triangleq T(V) - S \). The distribution of \( L_2 \) is obtained as follows

\[
\begin{align*}
 f_{L_2}(l_2) &= f_T(v) * f_S(-s) \\
 &= e^{-\frac{l_2^2}{2\sigma_v^2 + \sigma_s^2}} \int_{-\infty}^{\infty} e^{-\frac{(v-m_{l_2})^2}{2\sigma_s^2}} dv \\
 &\quad + Q_{\sigma_2^2}(\frac{\Delta}{\Delta_s}) \left( e^{-\frac{(l_2+\Delta/\Delta_s)^2}{2\sigma_s^2}} + e^{-\frac{(l_2-\Delta/\Delta_s)^2}{2\sigma_s^2}} \right),
\end{align*}
\]

where \( Q_{\sigma_2^2}(x) \triangleq \int_{x}^\infty e^{-\frac{t^2}{2\sigma_s^2}} dt \), \( \Delta \triangleq \Delta_s/\Delta \), and \( m_{l_2}, \sigma_{l_2} \) are defined as below:

\[
\begin{align*}
 m_{l_2} &\triangleq \frac{\alpha^2 \sigma_s^2 l_2}{\sigma_s^2 + \alpha^2 \sigma_s^2} \\
 \sigma_{l_2} &\triangleq \frac{\alpha \sigma_s}{\sqrt{\sigma_s^2 + \alpha^2 \sigma_s^2}}.
\end{align*}
\]

The cumulative distribution function (cdf) of \( X \) is obtained as follows:

\[
F_X(x) = \begin{cases} 
1, & \text{if } x \geq \frac{\Delta}{2}, \\
\sum_{i=-\infty}^{\left\lfloor \frac{\Delta}{2\Delta} \right\rfloor} f_{L_2}(t) dt, & \text{if } -\frac{\Delta}{2} \leq x < \frac{\Delta}{2}, \\
0, & \text{if } x \leq -\frac{\Delta}{2}.
\end{cases}
\]

Then, \( f_X(x) \) is obtained by differentiating (16), and is given in (23). Notice that in 1DL, \( X \) is limited to \([-\Delta/2, \Delta/2]\). The quantization step size, \( \Delta \), must be chosen such that the channel input power constraint is satisfied. Similarly to the ICO scheme, this will be done numerically.

The received signal at the receiver can be written as

\[
y = x + s + w = (T(v) - s) \mod \Delta + s + w = -\lfloor T(v) - s - (T(v) - s) \mod \Delta \rfloor + T(v) + w = -Q(T(v) - s) + T(v) + w.
\]

At the receiver, decoding is done based on MMSE estimation. The source is reconstructed as in (12), where in this case \( q_i \)'s are the points to which the term \( T(v) - s \) is concentrated. The final distortion is obtained in a similar manner to (13). The performance of this transmission scheme, and other features will also be discussed in Section IV together with others.

E. Interference Concentration with Non-Uniform Quantizer (ICO-NU)

Note that both the ICO and the 1DL schemes give some shape to the interference, rather than simply reducing its variance. Both schemes use uniform quantization for this. In this section we consider using a non-uniform quantizer for the ICO scheme, which will allow using different companders for the source sample depending on the interference signal.

In classical scalar quantization, non-uniform quantization is employed in order to reduce the quantization noise for the more likely values of the underlying signal. With such a quantizer, in our setting, we would have smaller intervals around zero, and the interval size would increase as we go further away from the origin. Note that, this would reduce the transmission power allocated for interference concentration. However, this would also mean that we have to compress the source signal even further when the interference realization is close to zero. We observe that the final distortion benefits more from increasing \( \Delta \); that is, having quantization points with larger separation. Hence, we apply the opposite of typical scalar quantization, and use a lower resolution quantization for more likely values of the interference, and decrease the quantization interval size as we go further away from zero.

As before, the interference signal is concentrated to the middle point of the quantization interval into which it falls. Since the length of the quantization interval depends on the realization of the interference, a different compander function will be used for each interval. Quantization intervals are defined as follows

\[
\xi_i \triangleq \{ s : |s| \leq \frac{\Delta_i}{2} \},
\]

\[
\xi_i \triangleq \{ s : B_i \leq |s| \leq B_i + \Delta_i \}, \quad i = \{ 2, 3, \ldots \},
\]

where

\[
B_i \triangleq \begin{cases} 
\frac{\Delta_i}{2}, & \text{if } i = 2, \\
\frac{\Delta_i}{2} + \sum_{j=1}^{i-1} \Delta_j, & \text{if } i > 2.
\end{cases}
\]

We define the \( \bar{f}_S(s) \) function as follows

\[
\bar{f}_S(s) \triangleq \frac{1}{f_S(s)^a},
\]

where \( f_S(s) \) is the pdf of the interference \( S \), and \( a > 0 \) is a parameter to be determined. The quantization points \( q_i \) for this non-uniform quantizer are the mid-points of the quantization intervals \( \xi_i \) for \( i = 1, 2, \ldots \).

The length of the \( i \)-th quantization interval, \( \Delta_i \), is chosen such that

\[
2 \int_{s \in \xi_i} \bar{f}_S(s) ds = \int_{s \in \xi_i} \bar{f}_S(s) ds, \quad \forall i.
\]
The Shannon theoretic lower bound obtained by evaluating the proposed novel transmission schemes. We will also include the Shannon theoretic lower bound obtained by evaluating the rate-distortion function of the Gaussian source at the capacity of the underlying channel when the interference is completely removed. Not surprisingly this lower bound is quite loose in general.

In Figures 2 to Figure 5, performances of the four proposed transmission schemes are shown and compared to the lower bound for different interference and noise variances. The source variance is fixed to $\sigma_s^2 = 1$ in all the figures. Since simulating ICO-NU is computationally intensive, we have included it only in Figures 2 and 3.

We observe that ICA outperforms other schemes in Figure 2 which corresponds to the low interference and high noise scenario. In all other cases, structured interference cancellation schemes outperform ICA.

We also observe that in all the simulation results, 1DL outperforms ICO, even though the performances of the two schemes have relatively similar behavior. In Figure 6, the size of the quantization interval $\Delta$ for both ICO and 1DL is plotted against the power constraint. As it is seen in this figure, for all power constraint values 1DL uses a larger $\Delta$ than ICO for quantization, which explains the improved performance of 1DL compared to ICO, since a larger $\Delta$ means that the source is compressed less, and hence, can be reconstructed with a smaller distortion on average.

For ICO-NU we optimize (12) and (13) over $\Delta$ and $\Delta_s$ as well as $a$. As it can be seen in Figures 2 and 4, ICO-NU improves the performance of the ICO scheme, and even outperforms 1DL in both the low and high noise regimes. In our future work we will extend the non-uniform quantization idea to the 1DL scheme which can further lower the average distortion, and close the gap to the lower bound. However, we believe that the theoretical lower bound is loose in general (especially in the high interference regime), and identifying a better lower bound will be instrumental in characterizing the performance limits in this problem.

IV. NUMERICAL RESULTS

We remark here that obtaining closed-form expressions for the optimal performance of JSCC under strict delay constraints is extremely difficult if not impossible. Instead, in this section, we provide numerical results comparing the performances of the proposed novel transmission schemes. We will also include the Shannon theoretic lower bound obtained by evaluating the

\begin{align*}
\text{For Gaussian interference we it can show that} \quad i > j \Rightarrow \Delta_i \leq \Delta_j. \\
\text{At the transmitter, if } S \text{ falls into the quantization interval } \xi_i, \text{ source } V \text{ is transformed as follows} \\
T(v, s) = \begin{cases} \\
\Delta_i v, & \text{if } v \geq \Delta_i, s \in \xi_i \\
\Delta_i v, & \text{if } -\Delta_i \leq v < \Delta_i, s \in \xi_i \\
-\Delta_i, & \text{if } v < -\Delta_i, s \in \xi_i \\
\end{cases}
\end{align*}

where $T(v, s)$ denotes the companding function in order to highlight its dependence on the realization of the interference. The transmitted signal is generated as below

\begin{align*}
x = T(v, s) - (s \mod \Delta),
\end{align*}

where $(s \mod \Delta)$ denotes the quantization noise for the non-uniform scalar quantizer with quantization points $q_i$ and decision regions $[q_i - \Delta_i/2, q_i + \Delta_i/2]$.

To satisfy the average power constraint we follow the same approach as in Section III-B. We have

\begin{align*}
\mathbb{E}[X^2] = \int_{-\Delta}^{\Delta} x^2 \sum_{j=1}^{\infty} f_X(x|\xi_j) P_r(\xi_j) \leq P. 
\end{align*}

Solving (26) requires the distribution of $S \mod \Delta$ for the non-uniform quantizer. We define $U \triangleq S \mod \Delta$. Since $\max\{\Delta_i\} = \Delta_1$, we have $U \in [-\Delta_1, \Delta_1]$. As before, first the cdf of $U$ is found, then by differentiating, we obtain $f_U(u)$.

The received signal for the ICO-NU scheme is given by

\begin{align*}
y & = x + s + w \\
& = T(v, s) - (s \mod \Delta) + s + w \\
& = T(v, s) + q_i + w.
\end{align*}

We use MMSE estimation as in (12) to estimate the transmitted source.

V. CONCLUSIONS

In this paper we have studied the problem of zero-delay transmission of a Gaussian source over an AWGN channel in the presence of known interference at the transmitter. Due to the zero-delay constraint and the memoryless nature of the source samples and the interference signals over time, causal
and non-causal availability of the interference information are equivalent in this setting. We have proposed one linear and three non-linear zero-delay JSCC schemes. The linear scheme is based on interference cancellation, whereas the non-linear schemes shape the interference and convert it into structured interference, and use companding for the transmission of the source samples.

We have shown that the proposed non-linear coding schemes can achieve zero-distortion in the limit of zero noise, whereas this is not possible through the linear ICA scheme when the interference is strong. We have also introduced the novel idea of non-uniform interference concentration for this problem, and have shown that the corresponding ICO-NU scheme achieves the best performance among the studied transmission techniques.

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